has the effect of reducing the electrical power input for a given output, thereby raising the efficiency and lowering the specific power.

The specific impulse, specific power, and power efficiency are all functions of the stored energy per plasma slug mass E and are shown in Fig. 3. The relations between η and I_{sp} and P/F and I_{sp} are shown in Fig. 4. The effect of reducing R_0 is seen here to be very marked. The relations between η, P/F, and I_{sp} do not vary with L_0.

The average thrust is shown in Fig. 5 as a function of E. For a constant V_0, $\bar{F}$ increases with increasing m (decreasing E), and, for a constant m, $\bar{F}$ increases with V_0 (increasing E). Since η increases with increasing E, the most advantageous method of increasing the average thrust during operation is to increase the charging voltage.

The total impulse per pulse, equal to ΔV multiplied by the total space vehicle mass, is a linear function of the electrical power input. It is a more complex function of m, as is seen in Fig. 6.

On the basis of these results, the rail accelerator appears to be useful for propulsion in space. The range of operation, where the efficiency is neither too low nor the specific power too high, appears to lie between 1500 and 5000-sec I_{sp}. The rail accelerator has the distinct advantage that one engine can be operated at various points in this range.

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Technical Comments

Comments on "Transition Correlations for Hypersonic Wakes"

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In a recent note,[1] Zeiberg has shown that the effects of body shape can be removed from a correlation of wind tunnel and ballistics range wake-transition data for various two-dimensional and axisymmetric models by using the Reynolds number based on freestream properties and the distance from the model base to the transition location, a "body bluntness" parameter, and the freestream Mach number. The authors have correlated the same small-scale data[2,3] as have many others,[4–6] each interpreting the results in a different manner but all having essentially the same goals, namely, to understand the phenomenon and to derive a simple formula for the prediction of wake transition for full-scale hypersonic re-entry conditions. Although a correlation such as suggested by Zeiberg, which employs concepts carried over from successful boundary-layer and free-shear-layer transition correlations, does in fact satisfy the latter goal, it contains implications that are contrary to the present understanding of the phenomenon, and its use to predict transition at conditions outside the range of small-scale experimental conditions is certainly questionable.

It is well established that the distance to transition in the wake is a function of the local Mach number and "some" Reynolds number (almost any will correlate the data for a given model). A close scrutiny of the small-scale data at a given freestream Mach number and model shape reveals the behavior sketched in Fig. 1.[4,5] The concept of a unique transition Reynolds number ($\rho_e U_e x_{TR}/\mu_e$) appears to be valid only within a limited range of body Reynolds numbers and/or unit Reynolds numbers (see Table 1). At lower body Reynolds numbers, the transition distance moves out rapidly toward infinity (which has been explained by Lees[4] on the basis of energy arguments), whereas at very high body Reynolds numbers, the transition distance "slows down" as it

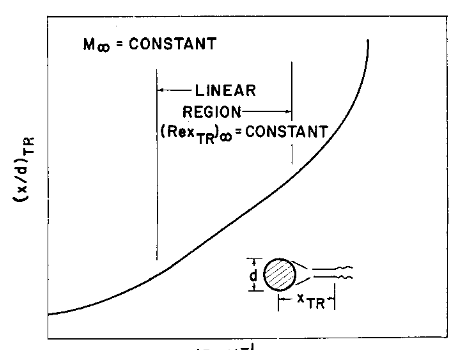

Fig. 1 Schematic representation of wake transition distance vs freestream Reynolds number.

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Table 1 Small-scale wake transition data

Bar, see Fig. 2	Model type	Semivertex angle, deg	Diameter or base height, in.	$(Re/\text{in.})_\infty \times 10^{-5}$	$(Red)_\infty \times 10^{-5}$	$(Rex_{TR})_\infty \times 10^{-5}$	$(x/d)_{TR}$	Ref.
1	Cone	12.5	0.1875–0.375	0.07–2.2	0.026–0.23	0.7–1.6	3–30	6
2	Sphere	...	0.25	0.42–3.2	0.10–0.80	1.4–2.7	2–25	7
3	Cone	12.5	0.1875–0.375	0.4–4.7	0.026–0.89	2.7–5.3	5–42	6
4	Wedges	2.5–22.5	0.088 –0.30[a]	0.7–2.4	0.091–0.72	3.0–10.0	6–74	5
5	Cylinder	...	0.10–0.30	0.74–2.4	0.10–0.45	1.6–6.3	4–56	8
6	Sphere	...	0.125–0.500	1.2–11.5	0.29–3.0	2.4–7.2	2–25	7
7	Sphere	...	0.125–0.500	0.43–2.3	0.21–0.45	17.4–28.5	54–100	7
8	Cone	10–15	0.3–0.4	2.5–27.5	0.85–8.5	9.9–150	4–22	2
9	Sphere	...	0.25	1.7–6.9	0.43–1.7	7.8–15.0	5–35	7

[a] The effective base height d for the $2\frac{1}{2}°$ slender wedges corresponds to the actual base height plus the viscous-displacement effect on the wedge surfaces.

approaches the wake neck (i.e., the rear stagnation point). The latter effect is often referred to as "sticking."‡ The ranges of unit Reynolds numbers, body Reynolds numbers, and model size and geometry which define each of these regions are evident in some of the small-scale experiments. However the scaling laws for extrapolating these regions have not yet been established.

The "unified wake transition correlation" of Zeiberg (Fig. 4, Ref. 1) is repeated here as Fig. 2 with some of the data that occur outside the "linear region" of Fig. 1 included. The "scatter" is as much as an order of magnitude. The range of conditions over which the data were obtained is indicated in Table 1.

The correlation suggested by Pallone, Erdos, and Eckerman,[2] for example, employs a Reynolds number based on the relative wake velocity $(1 - U_c/U_e)$ and the wake diameter. This does not reduce the scatter in a conclusive manner but does embody a significant dependence of transition distance on body size at low unit Reynolds numbers. A comparison of the results of this correlation and that of Zeiberg for a sharp cone at 22,000 fps is given in Fig. 3, where a value of $(Rex_{TR})_\infty = 10^8$ at $M_\infty = 22$ has been assumed as a reasonable extrapolation from Fig. 2. Although the results are essentially in agreement below 100,000 ft (and this could be improved by adjusting the assumed value of Rex_{TR}), it is important to note that 1) there is no effect of body size in the prediction based on Zeiberg's correlation (i.e., $x_{TR} \simeq 150$ ft at 125,000 ft for a 15-ft-long vehicle or a 0.15-ft model); 2) order-of-magnitude differences in the predicted transition distances (thousands of feet) occur at 200,000 ft, for example, regardless of the fact that both methods are based on correlations of the same experimental data; and 3) the method of Pallone, et al.,[2] indicates a completely laminar wake at some altitude, whereas Zeiberg's method always predicts a finite x_{TR} for a finite unit Reynolds number.

Hopefully, further theoretical work on the dynamical stability of wakes along the lines pursued by Gold[9] and Kronauer[10] will lend more insight to the problem and guide in the selection of the appropriate parameters for a correlation of wake-transition data. In the meantime, however, simplified correlations of the data must be carefully scrutinized to determine their known range of validity and their implications when extended beyond this range.

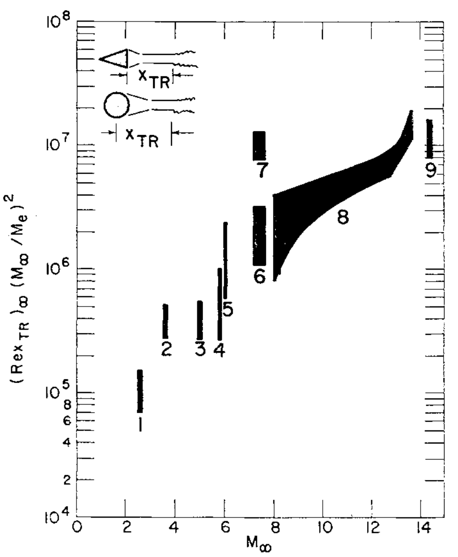

Fig. 2 Unified wake transition correlation.

‡ For example, Demetriades[5] defines the "sticking distance" as the apparent intercept of the wake transition curve (Fig. 1) with the $1/(Red)_\infty$ axis [i.e., extrapolation of the data to $(Red)_\infty \to \infty$]. There is, however, no physical evidence of the transition point even having "gotten stuck" prior to reaching the wake neck.

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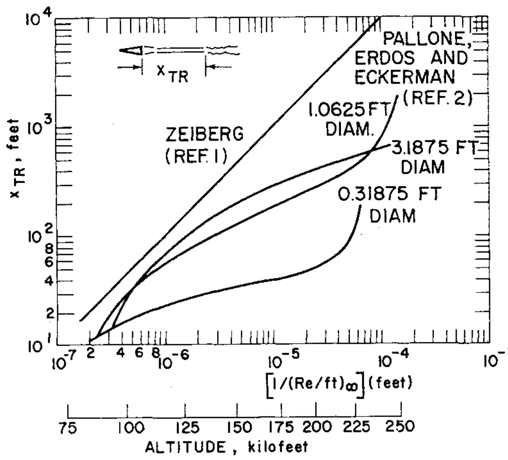

Fig. 3 Wake transition predictions for 12° cone at 22,000 fps.

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Reply by Author to J. I. Erdos and H. Gold

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ERDOS and Gold, in the preceding comment, question the value of a simplified correlation of wake transition data[1]; however, the fact remains that the available ballistic range data are inconclusive with regard to model size dependence,[1,2–6] and small differences in body shape.[1,2,5] Thus, an involved correlation scheme based upon these data does not seem to be indicated at present. The behavior shown schematically in Fig. 1 of the foregoing comment is not demonstrated by the data (with the possible exception of a hint at the shape by two points of the sphere data[8,7] at $M_\infty \simeq 7.5$, i.e., region 7 on Fig. 2 of the preceding note) even though the trend may be implied by the initial results of stability investigations. Therefore, when using the data outside of the range of the experiments, one must be careful to avoid reading too much from the data; e.g., note the reversals in the effect of body size over the Reynolds number range for the cases shown in Fig. 3 of the preceding note (also see Fig. 1).

With regard to the comparison of correlations shown in Fig. 3 of the preceding comment, the author notes that, according to the curve-fit described in Ref. 9, the extrapolation of the "unified transition correlation" (Fig. 4 of Ref. 1 or Fig. 2 of Erdos and Gold's note) to $M_\infty = 22$ should indicate $(Re_{x_{TR}})^2_\infty\ (M_\infty/M_e) = 10^8$. For a 12° cone at $M_\infty = 22$, the quantity M_e (as defined by the author in Ref. 1) is about 14.5; then $(Re_{x_{TR}})_\infty \simeq 4.3(10)^7$, and the comparison of the author's correlation and that of Ref. 2 appears as shown in Fig. 1. It is seen that, for the larger bodies, agreement to within a factor of 3 (which is within the experimental data scatter according to all available correlations) is obtained for altitudes of 200 kft and below.

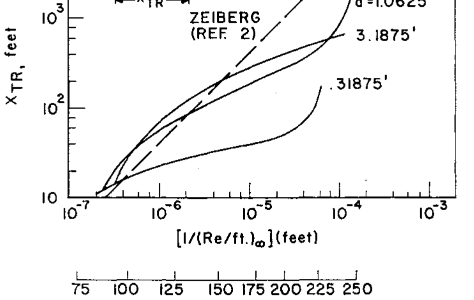

Fig. 1 Wake transition predictions for 12° cone at 22,000 fps.

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Comment on "Derivation of Element Stiffness Matrices"

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REFERENCES 1 and 2 describe methods of structural analysis in which the deflections of an element are expressed in terms of a number $N = n + l$ basic displacement functions. The element is loaded and/or attached to other elements at its nodes, which have n slopes and deflections. The difference between this and previous work is that l, the number of surplus shape functions, was previously assumed zero.

A comparison of Refs. 1 and 2 is of interest in that the solutions are independent, are expressed differently, and have different motivation. Pian remarks that by taking a large number l of surplus undetermined coefficients α, the equilibrium conditions are improved. Reference 1 suggests that the first few α, say $\alpha_1 \ldots \alpha_r$, may represent the necessary rigid body motions; this guarantees exact equilibrium. Experience with large l in a two-dimensional problem indicates that the difficulties of imposing conformity of slopes and deflections between elements increase with l.

In the $[G]$ of Ref. 2, the first r rows and columns will be zero because the rigid body motions do not contribute to the strain energy. Some arithmetic may then be saved by using only the nonzero terms of $[G]$, say $[G_0]$, a $(n - r) \times (n - r)$ matrix. Because $\alpha_1 \ldots \alpha_r$ are of no subsequent interest, the first r rows of $[B_a^{-1}]$ may be discarded leaving $[C_0]$, say, a $(n - r) \times l$ matrix. Then (8) of Ref. 2 becomes

$$\{\alpha^*\} = \begin{Bmatrix} \alpha^* \\ \alpha_b \end{Bmatrix} = \left[\begin{array}{c:c} C_0 & -C_0B_b \\ (n-r)\times n & (n-r)\times l \\ \hdashline 0 & I \\ l \times n & l \times l \end{array}\right] \begin{Bmatrix} q \\ \alpha_b \end{Bmatrix} = [M^*]\begin{Bmatrix} q \\ \alpha_b \end{Bmatrix}$$

where the asterisk means that the first r rows are missing.

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